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and prime to each other, there is in general no solution. So, we have only to find how two numbers not prime to each other may be obtained to fulfill the requirements

(C). Let n and p be two positive integers prime to each other, then (4), $n^3 - p^3 = hr^2$, and (5), $n^2 - p^2 = ks^3$, where r, s, h, k , are positive integers. Let m be a factor such that if n and p are both multiplied by it, they will fulfill the required condition. Then, (6), $m^3 n^3 - m^3 p^3 = m^3 hr^2 =$ a square. (7), $m^2 n^2 - m^2 p^2 = m^2 ks^3 =$ a cube. The first will be a square if $m = hc^2$ (8), and the second a cube if $m = kc^3$ (9). Combining (8) and (9), $c = \frac{h}{k}$ and $m = \frac{h^3}{k^2}$ (10).

Then by taking any two positive integers prime to each other, substituting, and finding values of h and k in equations (4) and (5), and multiplying each member by the value of m from (10), we have two numbers which fulfill the requirements. There is nothing in above proof to limit the method to positive integers prime to each other, but these may be taken as the basis of all positive numbers. When one of the numbers taken is negative, different results are derived. When both are negative, the value derived for m will change the numbers so that the difference of the cubes will not be negative. (Ex. 1) $5^3 - 3^3 = 98 = 2 \times 7^2$, $5^2 - 3^2 = 16 = 2 \times 2^3$, $h=2, k=2, m=2$. \therefore numbers are 10 and 6. (Ex. 2). $2^3 - 1^3 = 7$, $2^2 - 1^2 = 3$, $h=7, k=3, m = \frac{7^3}{3^2}$. \therefore numbers are $\frac{686}{9}$ and $\frac{343}{9}$. (Ex. 3).

$2^3 - (-1)^3 = 9$, $2^2 - (-1)^2 = 3$, $h=1, k=3, m = \frac{1}{9}$. \therefore numbers are $\frac{2}{9}$ and $-\frac{1}{9}$.

(Ex. 4). $1^3 - 0^3 = 1$, $1^2 - 0^2 = 1$, $h=1, k=1, m=1$. \therefore numbers are 1 and 0, as derived before in (A).

Also solved by *H. W. Draughon, A. L. Foote, P. H. Philbrick, and G. B. M. Zerr.*

3. Proposed by O. S. KIBLER, Superintendent of Schools, West Middleburg, Logan County, Ohio.

It is required to find three whole numbers in an arithmetical progression, such that the sum of every two of them shall be a square.

Solution by A. L. FOOTE, 80 Broad St., New York City.

Let $\frac{1}{2}x^2 - y$, $\frac{1}{2}x^2$, and $\frac{1}{2}x^2 + y$ be the numbers. Then we must have x^2 , $x^2 + y$, and $x^2 - y$ each squares and since x^2 is a square, we require only to make $x^2 + y$ and $x^2 - y$ squares. Let $x^2 + y = m^2$ and $x^2 - y = n^2$. Let $y = 2sx + s^2$ and we have $x^2 + y = x^2 + 2sx + s^2 = (x+s)^2$, and so we have but to find $x^2 - 2sx - s^2 = n^2$. Now let $x^2 - 2sx - s^2 = (x-m)^2 = x^2 - 2mx + m^2$ and we have $x = \frac{m^2 + s^2}{2m - 2s}$

where m and s may be assumed at pleasure. Let $m=5$ and $s=4$, then

$x = \frac{25+16}{10-8} = \frac{41}{2}$ and $\frac{1}{2}x^2 = \frac{1681}{8}$, $y = 2sx + s^2 = 164 + 16 = 180$, so we have $\frac{1681}{8}$, $\frac{241}{8}$, and $\frac{3121}{8}$. To render these integral multiply each by 16 and we have

482, 3362 and 6242 for the required numbers. The squares are $3844 = 62^2$, $6724 = 82^2$, and $9604 = 98^2$. The values of m must be so chosen that $\frac{1}{2}x^2 - y$ will be positive.

J. H. Drummond finds 380, 8450 and 16514. Also solved by P. S. Berg and H. W. Draughon.